distributions for the ideal gas and the real gas case. The temperature reaches above 23,000°K with a perfect gas assumption while the real gas effect causes the peak temperature to drop to approximately 16,000°K. As shown in Fig. 3, the pressure drops from the initial value of three atm to about 1.7 atm at the throat. The absorption coefficient k_ν which is the function of density and temperature has two distinct profiles for the two cases. Once the absorption coefficient is known, we can transform the solution into the real space x by Eq. (3). As shown in Fig. 4, the temperature distributions are somewhat different due to the transformation and the throat is located at about $x=3$ cm. With the known thermodynamic conditions in the nozzle, the heat transfer characteristics were then assessed.[12]

Finally, we can proceed to calculate the nozzle contours in the physical space. As one may notice the initial velocity is a function of A^*/A_i where A^* is the throat cross-section area. For the example case, the propagation velocity of 1.09×10^5 cm/sec requires the nozzle contour to have an area ratio $A^*/A_i=0.23$. In addition, for a known laser power, i.e., 100 kW, the initial cross-sectional area A_i and hence the complete nozzle size can now be determined (Fig. 4).

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Entrance Flow in a MHD Channel with Hall and Ion Slip Currents

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THE problem of entry flow in channels has been solved by a number of authors. Targ[1] has analyzed this problem by linearizing the inertia terms with the mean velocity. Sparrow et al.[2] improved Targ's method by introducing a stretched coordinate in the flow direction and obtained a closed-form solution for hydrodynamic flows. Using this method, Snyder[3] has analyzed MHD flows in the entrance region of a rectangular channel when the electric current obeys simple Ohm's law without Hall and ion slip currents. He also has given a good bibliography of the earlier work. Recently Chen and Chen[4] and Hwang[5] have considered the entry flow with an arbitrary inlet velocity profile. These studies of entry flow in channels are needed for operational MHD devices like power generators and MHD accelerators. But in these devices, usually the conducting material is a partially ionized gas and, therefore, the Hall and ion slip currents are important for these applications. Saric and Touryan[6] have considered the effect of these currents, using momentum integral method.

In the present analysis, the problem of two-dimensional MHD flow in the entrance region of a rectangular channel is solved with generalized Ohm's law under externally applied, electric field loading conditions. The magnetic Reynolds number is assumed to be small and the induced magnetic field is neglected.

Following the model suggested by Snyder,[3] the equations of motion in the present case are[7]

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{\sigma B_0^2}{\rho\sigma_0^2}\left[(1+\beta_e\beta_i)\left(\frac{E_y}{B_0}-u\right)+\beta_e\left(\frac{E_x}{B_0}+v\right)\right] \quad (1)$$

$$u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}+\frac{\sigma B_0^2}{\rho\sigma_0^2}\left[\beta_e\left(\frac{E_y}{B_0}-u\right)-(1+\beta_e\beta_i)\times\left(\frac{E_x}{B_0}+v\right)\right] \quad (2)$$

where

$$\sigma_0^2=(1+\beta_e\beta_i)^2+\beta_e^2$$

Multiplying Eq. (2) by $i=(-1)^{1/2}$ and adding Eq. (1), it becomes

$$u\frac{\partial u'}{\partial x}+w\frac{\partial u'}{\partial z}=-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)+\nu\frac{\partial^2 u'}{\partial z^2}-\frac{\sigma B_0^2\alpha_0}{\rho\sigma_0^2}\left(\frac{iE'}{B_0}+u'\right) \quad (3)$$

where $u'=u+iv$, $E'=E_x+iE_y$, and $\alpha_0=(1+\beta_e\beta_i)+i\beta_e$. Using the method of Sparrow et al.[2] to linearize the inertia terms by the mean velocity U and a weighting function $\epsilon(x)$, Eq. (3) is transformed as

$$\epsilon(x)U\frac{\partial u'}{\partial x}=\Lambda(x)+\nu\frac{\partial^2 u'}{\partial z^2}-\frac{\sigma B_0^2\alpha_0}{\rho\sigma_0^2}\left(\frac{iE'}{B_0}+u'\right) \quad (4)$$

where

$$U=\frac{1}{2h}\int_{-h}^{h}u\,dz$$

and

$$\frac{1}{2h}\int_{-h}^{h}v\,dz=0$$

Here h is half the height of the channel. Equation (4) is the same as Eq. (2) of Ref. 3, except that here the functions are

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complex functions. Repeating Snyder's analysis and using the following transformations to nondimensionalize,

$$Q=\frac{u'}{U},\ \xi=\frac{1}{R_e}\frac{x^*}{h},\ \eta=\frac{z}{h}, dx=\epsilon dx^*$$

$$R_e=\frac{\rho U h}{\mu},\ H_a=B_0 h(\frac{\sigma}{\mu})^{1/2},$$

$$p'=\frac{2p}{\rho U^2},\ \text{and}\ H_0^2=\frac{H_a^2\alpha_0}{\sigma_0^2} \tag{5}$$

Eq. (4) becomes

$$\frac{\partial Q}{\partial \xi}=\frac{\partial^2 Q}{\partial \eta^2}-\frac{1}{2}(\frac{\partial Q}{\partial \eta})_{\eta=-1}^{\eta=+1}+H_0^2(1-Q) \tag{6}$$

The boundary conditions are

$$Q=3/2(1-\eta^2)\quad \text{at}\ \xi=0$$

$$Q=0\qquad \text{at}\ \eta=\pm 1 \tag{7}$$

Using the technique of Laplace transform or eigenfunction expansion, $Q(\xi,\eta)$ is obtained as

$$Q(\xi,\eta)=H_0(\frac{\cosh H_0\eta-\cosh H_0}{\sinh H_0-H_0\cosh H_0})+2\sum_{n=1}^{\infty}(\frac{1}{\gamma_n^2+H_0^2}-\frac{1}{\gamma_n^2})\times(\frac{\cos\gamma_n\eta}{\cos\gamma_n}-1)\exp\left\{-(\gamma_n^2+H_0^2)\right\} \tag{8}$$

where $\gamma_n (n=1,2,3,...)$ are roots of $\gamma=\tan\gamma$. Hence, the velocity components are

$$q_1(\xi,\eta)=(A_1\cosh a\eta\cos b\eta+B_1\sinh a\eta\sin b\eta+C_1)/D+2\sum_{n=1}^{\infty}F_nG_n' \tag{9}$$

$$q_2(\xi,\eta)=(A_1\sinh a\eta\sin b\eta-B_1\cosh a\eta\cos b\eta+C_2)/D-2\sum_{n=1}^{\infty}F_nG_n'' \tag{10}$$

$A_1=a\sinh a\cos b-(a^2+b^2)\cosh a\cos b+b\cos a\sin b$

$B_1=a\cosh a\sin b-(a^2+b^2)\sinh a\sin b-b\sinh a\cos b$

$C_1=(a^2+b^2)(\sinh^2 a+\cos^2 b)-a\cosh a\sinh a-b\cos b\sin b$

$C_2=a\sin b\cos b-b\sinh a\cosh a$

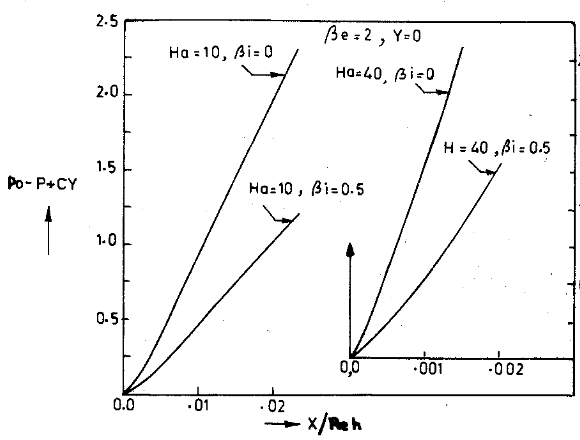

Fig. 1 Entrance region pressure distribution.

$D=(\sinh^2 a+\sin^2 b)+(a^2+b^2)(\sinh^2 a+\cos^2 b)-2a\sinh a\cosh a-2b\sin b\cos b$

$$F_n=[\frac{\cos\gamma_n\eta}{\cos\gamma_n}-1]\exp\left[-(\gamma_n^2+\alpha_1^2)\right]$$

$$G_n'=\frac{(\gamma_n^2+\alpha_1^2)\cos\alpha_2^2\xi-\alpha_2^2\sin\alpha_2^2\xi}{(\gamma_n^2+\alpha_1^2)^2+\alpha_2^4}-\frac{\cos\alpha_2^2\xi}{\gamma_n^2}$$

$$G_n''=\frac{(\gamma_n^2+\alpha_1^2)\sin\alpha_2^2\xi+\alpha_2^2\cos\alpha_2^2\xi}{(\gamma_n^2+\alpha_1^2)^2+\alpha_2^4}-\frac{\sin\alpha_2^2\xi}{\gamma_n^2}$$

$$a^2=\frac{H_a^2}{2\sigma_0^2}[\sigma_0+(1+\beta_e\beta_i)]$$

$$b^2=\frac{H_a^2}{2\sigma_0^2}[\sigma_0-(1+\beta_e\beta_i)]$$

$$\alpha_1^2=\frac{H_a^2(1+\beta_e\beta_i)}{\sigma_0^2}$$

$$\alpha_2^2=\frac{H_a^2\beta_e}{\sigma_0^2}$$

In Eqs. (9) and (10), the first parentheses give the fully developed components as obtained by Zhaveri[8] and Mittal and Bhat.[9] The terms with summation sign give the correction for the entrance region. ϵ is obtained by equating the pressure gradients from the momentum equation and from the mechanical energy equation, as

$$\epsilon(H_\alpha,\xi)=\frac{\frac{3}{2}\int_0^1 q_1^2\frac{\partial q_1}{\partial\xi}d\eta}{\int_0^1 q_1\frac{\partial q_1}{\partial\xi}d\eta}-2 \tag{11}$$

The pressure distribution is obtained by integrating Eq. (1) and (2) over the cross section of the channel. In dimensionless form this gives

$$p'(\xi,Y')=p_0'-2[\alpha_1^2(1-\frac{E_y}{B_0U})-\alpha_2^2\frac{E_x}{B_0U}+(\frac{dq_{1f}}{d\eta})_{\eta=1}]\times\int_0^\xi\epsilon d\xi+2[\int_0^\xi\epsilon(\frac{\partial(q_1-q_{1f})}{\partial\eta})_{\eta=1}d\xi+1.2-\int_0^1 q_1^2 d\eta]-CY' \tag{12}$$

where

$$C=\frac{2}{R_e}[\alpha_2^2(1-\frac{E_y}{B_0U})+\alpha_{02}^2\frac{E_x}{B_0U}-\frac{dq_{2f}}{d\eta}\Big|_{\eta=1}]$$

As is seen from Eqs. (9) and (10), the Hall and ion slip currents give some small disturbance to the velocity com-

Table 1 Comparison of entrance lengths and fully developed values of K

H_a	β_e	β_i	$X_{5\%}$	K_{fd}
10	0	0	0.0143[a]	−0.353[a]
10	2	0	0.0207	−0.2978
10	2	0.5	0.0216	−0.2622
40	2	0	0.0015	−0.3754
40	2	0.5	0.0202	−0.3718

[a]From Ref. 4.

ponents. Their amplitude is small and the period is larger than the entry length. These disturbances are not found in the absence of the Hall and ion slip currents. This increases the entry length. This conclusion agrees with the results of Saric and Touryan.[6]

Numerically, the entry length has been calculated on the basis of 5% deviations in the fully developed centerline velocity. The entry lengths and K_{fd}, the fully developed values of the correction term K due to entrance effects in the pressure distribution are shown in Table 1.

The entry length for the case of no Hall and ion slip currents with Hartmann number $H_a = 10$ was found to be 0.0143 $R_e h$ by Chen and Chen.[4] With the Hall parameter $\beta_e = 2.0$ it is 0.0207 $R_e h$ and with $\beta_e = 2.0$ and ionslip parameter $\beta_i = 0.5$, it is found to be 0.0216 $R_e h$. As Hartmann number increases the entrace length decreases sharply.

The pressure distribution in the entrance region is shown in Fig. 1 for different values of H_a, β_e, and β_i. It shows small curvature in the inlet region. For the downstream region it becomes constant. The solutions presented herein may be useful for the investigation of the stability of a developing MHD flow with Hall and ion slip currents.

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Correlation for Gasdynamic Laser Gain

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Introduction

GASDYNAMIC lasers (GDL's) have spearheaded a breakthrough in high energy laser technology. These devices are essentially supersonic wind tunnels which create a lasing medium by rapid expansion of a vibrationally excited molecular gas. Under suitable conditions, laser power can be extracted from the supersonic stream. The physical processes, practical importance, experimental data, and theoretical analyses associated with GDL's are described in a recent book.[1]

Theoretical prediction of gasdynamic laser performance generally requires a sophisticated nonequilibrium nozzle flow computer program, such as Ref. 2. Moreover, GDL performance is a function of numerous variables, such as nozzle shape and size, reservoir gas temperature and pressure, mixture ratio, etc. For these reasons, an analysis of gasdynamic laser characteristics is usually restricted to those individuals and organizations which have considerable computational capability. Therefore, there is a need for engineering correlations which allow quick and easy hand calculations of GDL performance without a gross compromise in accuracy. The purpose of the present Note is to provide such an engineering correlation. In particular, a formula is provided for the calculation of peak small-signal gain for CO_2-N_2-H_2O gasdynamic lasers. Such a mixture is common in practical GDL field and laboratory devices.

The Correlation

Small-signal gain, G_o, is a measure of the amplifying property of the laser medium such that $dI_v = G_o I_v dx$, where I_v is the laser beam intensity, and dI_v is the increase in intensity over a distance dx. Essentially, G_o is a negative absorption coefficient. In gasdynamic lasers, G_o varies with distance in the flow direction, first increasing, reaching a peak, then decreasing downstream.[1] In the present Note, the peak value of G_o is correlated as the following functional variation,

$$G_o = f(P_o, T_o, A_e/A^*, h^*, X_{N_2}, X_{CO_2}, X_{H_2O}) \qquad (1)$$

where P_o and T_o are the reservior gas pressure and temperature, respectively, A_e/A^* is the nozzle exit-to-throat area ratio, h^* is the throat height, and X_i is the mole fraction of species i in the mixture.

Using the computer program described in Ref. 2, with updated kinetic rates as given in Ref. 3, a large number of parametric variations were obtained. The parameters were those of Eq. (1), with the exception that P_o and h^* were grouped as the product $P_o h^*$, based on binary scaling, and $X_{CO_2} + X_{N_2} + X_{H_2O} = 1$. This large bulk of numerical data was then systematically correlated, using a combination of chi-square and Gaussian functions, aided by least-squares fits. The details are given in Ref. 4. The resulting engineering formula, although rather long, can easily be evaluated on a slide rule or pocket calculator. The formula is

$$G_o = K_O - G[(1-Q1) + (1-Q2) + (1-Q3)] - Q4 \qquad (2)$$

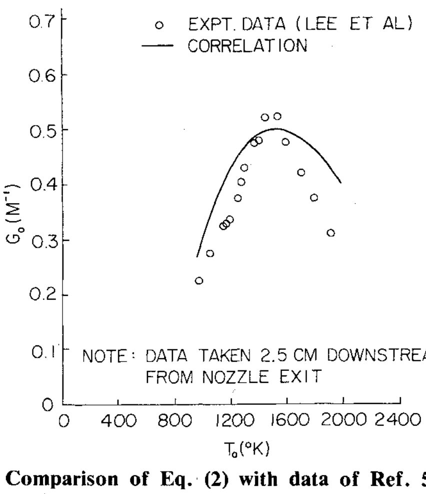

Fig. 1 Comparison of Eq. (2) with data of Ref. 5. $h^* = 0.1$ cm, $A_e/A^* = 15$ ($M = 4$), $P_o = 9$ atm, %$H_2O = 2.4$, %$CO_2 = 6.3$.

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